Solutions Manual for

Introduction to Ratemaking and Loss Reserving for Property and Casualty Insurance

FOURTH EDITION

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SOLUTIONS TO THE TEXTBOOK EXERCISES

Chapter 1

1.1 (a) The law of large numbers states that as the number of observations increases, the difference between the observed relative frequency of an event and the true underlying probability tends to zero.

(b) The risk to the insurance company is not equal to the sum of the individual risks (variance of total outcome) transferred to it.

1.2 $u(x) = k \cdot \log x$

$u'(x) = \frac{k}{x} = kx^{-1}$

$u''(x) = -kx^{-2}$

Since $u'(x) > 0$ and $u''(x) < 0$, this decision maker is risk averse.

1.3 To reflect the risk attribute, we use utility value rather than monetary value.

EUV(A) = $1(.6) + .5(.1) + 0(.3) = .65$
EUV(B) = $.9(.5) + .8(.3) + .2(.2) = .73$

A risk avoider would choose Proposal B.

1.4 (a) EMV(X) = $50{,}000(.35) - 20{,}000(.65) = 4500$
EMV(Y) = $5{,}000(.55) - 5{,}000(.45) = 500$

Both choose X based on expected monetary value.

(b) A: EUV(X) = $1.00(.35) + .30(.65) = .545$
EUV(Y) = $.55(.55) + .45(.45) = .505$

Businessman A chooses X based on expected utility value.

B: EUV(X) = $1.00(.35) + .55(.65) = .70750$
EUV(Y) = $.77(.55) + .709(.45) = .74255$

Businessman B chooses Y based on expected utility value.

1.5 (a) $u(P) = \sqrt{P-1000} = (P-1000)^{1/2}$

$u'(P) = \frac{1}{2}(P-1000)^{-1/2}$

$u''(P) = -\frac{1}{4}(P-1000)^{-3/2}$

Management is risk averse since $u'(P) > 0$ but $u''(P) < 0$.

(b) (i) $\text{EMV(A)} = 3000(.10) + 3500(.20) + \cdots + 5000(.10) = 4000$
$\text{EMV(B)} = 2000(.10) + 3000(.25) + \cdots + 6000(.10) = 4000$
The EMV is the same for both proposals, so management would be indifferent on this basis.

(ii) First we find the utility value of each profit amount.

Proposal A			Proposal B		
Profit	Utility	Probability	Profit	Utility	Probability
3,000	44.72	.10	2,000	31.62	.10
3,500	50.00	.20	3,000	44.72	.25
4,000	54.77	.40	4,000	54.77	.30
4,500	59.16	.20	5,000	63.24	.25
5,000	63.24	.10	6,000	70.71	.10

$$\begin{aligned}\text{EUV(A)} &= 44.72(.10) + 50.00(.20) + \cdots + 63.24(.10)\\ &= 54.536\\ \text{EUV(B)} &= 31.62(.10) + 44.72(.25) + \cdots + 70.71(.10)\\ &= 53.654\end{aligned}$$

Management chooses A based on expected utility value.
NOTE: We can also see that Proposal B has the larger variance.

1.6 (a) EMV (no insurance) = $10{,}000(p) + 30{,}000(1-p)$
EMV (insurance) = $20{,}000(p) + 25{,}000(1-p)$
Equating and solving for p we have

$$10{,}000p + 30{,}000(1-p) = 20{,}000(p) + 25{,}000(1-p),$$

which solves for $p = \frac{5}{15} = \frac{1}{3}$.

(b) The utility values of the various profit amounts are as follows.

	Freeze	No Freeze
No Insurance	71	158
Insurance	123	141

$$\text{EUV (no insurance)} = 71(p)+158(1-p)$$

$$\text{EUV (insurance)} = 123(p)+141(1-p)$$

Equating and solving for p we have

$$71p+158(1-p) = 123p+141(1-p),$$

which solves for $p=\frac{17}{69}\cong .2464$.

1.7 With insurance your utility position is $\left(\frac{50{,}000-G}{10{,}000}\right)^{.9}$.

Without insurance your expected utility position is

$$\int_0^{30{,}000}\left(\frac{50{,}000-x}{10{,}000}\right)^{.9}\left(\frac{1}{30{,}000}\right)dx$$

$$= -\left(\frac{1}{1.9}\right)\left(\frac{1}{10{,}000}\right)^{.9}\left(\frac{1}{30{,}000}\right)(50{,}000-x)^{1.9}\Big|_0^{30{,}000}$$

$$= \left(\frac{1}{1.9}\right)\left(\frac{1}{10{,}000}\right)^{.9}\left(\frac{1}{30{,}000}\right)\left[(50{,}000)^{1.9}-(20{,}000)^{1.9}\right].$$

Equate the two expected utility positions and solve for G.

$$(50{,}000-G)^{.9} = \left(\frac{1}{1.9}\right)\left(\frac{1}{30{,}000}\right)\left[(50{,}000)^{1.9}-(20{,}000)^{1.9}\right].$$

$$= 12{,}258.46$$

Then we have $G = 50{,}000-(12{,}258.46)^{10/9} = 15{,}109.54$.

1.8 With no wager your wealth is 20,000 and your expected utility position is $1-\exp\left(-\frac{20{,}000}{100{,}000}\right) = .1812692$.

If you wager an amount w, you end up with either $30{,}000-w$ (if you win) or $20{,}000-w$ (if you lose). The expected utility position is

$$\tfrac{1}{2}\left[1-\exp\left(-\frac{30{,}000-w}{100{,}000}\right)\right]+\tfrac{1}{2}\left[1-\exp\left(-\frac{20{,}000-w}{100{,}000}\right)\right].$$

Equating and solving for w we have

$$\begin{aligned}
.1812692 &= 1-\tfrac{1}{2}(.7408182)\cdot\exp\left(\tfrac{w}{100,000}\right) \\
&\qquad -\tfrac{1}{2}(.8187307)\cdot\exp\left(\tfrac{w}{100,000}\right) \\
.8187308 &= (.779774)\cdot\exp\left(\tfrac{w}{100,000}\right) \\
\exp\left(\tfrac{w}{100,000}\right) &= 1.049958 \\
w &= 4,875.05
\end{aligned}$$

1.9 With no wager the wealth is 3000, with expected utility value $10,000(3000)-(3000)^2 = 21,000,000$. With the wager the wealth is either 5000 (if she wins) or $3000-w$ (if she loses), with expected utility value

$$.30\left[10,000(5000)-(5000)^2\right]+.70\left[10,000(3000-w)-(3000-w)^2\right].$$

Equating and solving for w we have

$$21,000,000 = 7,500,000+.7[21,000,000-4000w-w^2],$$

which solves for

$$w = \frac{-2800\pm\sqrt{(2800)^2+4,800,000(.7)}}{2} = 390.46.$$

1.10 $$\begin{aligned}
\mu &= \tfrac{1}{3}(2000+4000+6000) = 4000 \\
\sigma^2 &= \tfrac{1}{3}\left[(2000-4000)^2+(4000-4000)^2+(6000-4000)^2\right] \\
&= \frac{8,000,000}{3} \\
\sigma &= \sqrt{\sigma^2} = 2000\sqrt{2/3} = 1632.99 \\
u &= \mu+\tfrac{\sigma}{6} = 4000+\frac{1632.99}{6} = 4272.17
\end{aligned}$$

The gross premium is 4500. Since this exceeds the expected utility loss, no insurance will be purchased.

1.11 (a) The gross premium is 1.10% of the expected loss, so we have

$$GP = 1.10 \cdot E(L)$$
$$= 1.10[10{,}000(.15) + 20{,}000(.04) + 50{,}000(.01)] = 3080$$

Utility with insurance $= U(525{,}000 - G) = 13.16527$

Utility without insurance

$$= .80u(525{,}000) + .15u(515{,}000)$$
$$+ .04u(505{,}000) + .01u(475{,}000) = 13.16571441$$

∴ Do not buy insurance.

(b) This time the company's gross premium is

$$GP = 1.10 \cdot E(L)$$
$$= 1.10[20{,}000(.04)(.50) + 50{,}000(.01)(.50)] = 715$$

Mr. Smith's expected utility position, with insurance, is $(G = 715)$

$$.80u(525{,}000-G) + .15u(525{,}000-10{,}000-G)$$
$$+ .04u(525{,}000-10{,}000-G)$$
$$+ .01u(525{,}000-25{,}000-G) = 13.16564312.$$

His expected utility position without insurance is 13.165714, from part (a).

∴ Do not buy insurance

1.12 The criteria to review (see Section 1.4) include the following:

1. Economically feasible (OK)
2. Economic value is calculable (common; should be)
3. Loss must be definite (OK)
4. Loss must be accidental (should be if no profit)
5. Exposures in risk class homogeneous (OK)
6. Units spatially and temporally independent (OK)

Yes, the insurance purchase is appropriate.

1.13 (a) This is really the same as Exercise 1.12; the risk is insurable.

(b) The net single premium is the expected value of the present value of the insurance payment. If Z denotes the random present value of payment, then

$$NSP = E[Z]$$

$$= 8000(1.10)^{-1}\left(\tfrac{1000}{7000}\right) + 5000(1.10)^{-2}\left(\tfrac{1500}{7000}\right) + 0 = 1924.44$$

(c) $Var(Z) = E[Z^2] - (E[Z])^2$

$$E[Z^2] = \left[8000(1.10)^{-1}\right]^2\left(\tfrac{10}{70}\right) + \left[5000(1.10)^{-2}\right]^2\left(\tfrac{15}{70}\right)$$

$$= 11{,}215{,}081$$

$$Var(Z) = 11{,}215{,}081 - 3{,}703{,}469 = 7{,}511{,}612$$

1.14 Gambling: Creates risk where none exists or needed to.
Takes dollars of high marginal utility.
If you win you get dollars of lower marginal utility (if risk averse).

Insurance: Transfers risk through pooling techniques.
Takes dollars of low marginal utility and protects dollars of high marginal utility. In total, society has higher total utility with insurance than without.

1.15 Risk: A measure of variation in economic outcomes
e.g.: risk of a monetary loss if house burns down

Peril: A cause of risk
e.g.: fire, wind, theft, illness

Hazard: A contributing factor to the peril
e.g.: poor wiring, location, moral hazard

Chapter 2

2.1 All-risks or comprehensive covers everything except what is specifically excluded. Specified perils only covers the named perils. All-risks cover will exclude several perils, such as nuclear radioactivity, war, wear and tear, so it is not absolutely all-risks.

2.2 (a) Salvage: Once the insurer has paid the policyholder full compensation for damaged property, it assumes ownership of the property and can sell it for its salvage value. This decreases the premium for the coverage.

(b) Subrogation: The insurer, having indemnified the policyholder, acquires the legal rights of the policyholder to sue the party at fault and recover costs. This will lower some premiums (e.g., homeowners dwelling coverage or auto collision), and raise the corresponding premium on the liability cover.

2.3 A loss is covered by a policy only if a covered peril is the proximate cause of a covered consequence (both are needed). A covered peril is the proximate cause if it is the cause that initiates an unbroken sequence of events leading to a covered consequence.

2.4 (a) Objectives of the coinsurance clause:

1. It encourages insurance to value.
2. It creates premium equity between insureds.
3. The overall rate level can be smaller but still adequate.

(b) Disadvantages of the coinsurance clause:

1. Not well understood by policyholder.
2. A policyholder who buys less than full coverage is only penalized if there is a claim, since he or she can pay a lower premium and get away with it.

3. The 80% coinsurance clause discriminates against those who carry higher levels of insurance.

4. Because of the misunderstanding of the coinsurance clause, some costly disputes arise over its use and meaning.

5. With high rates of inflation in real estate, a homeowner may unwittingly fall below the coinsurance percentage requirement.

6. A coinsurance percentage of less than 100% implies a recommendation to policyholders to buy less than full coverage.

2.5 Find X such that $X\left(\dfrac{400{,}000}{80\% \text{ of } 800{,}000}\right) = 320{,}000.$

The equation solves for $X = 512{,}000$.

2.6 Find X such that $\dfrac{120{,}000}{X\% \text{ of } 200{,}000} \cdot 10{,}000 = 7{,}500.$

The equation solves for $X = 80\%$.

2.7 The payment would be $\dfrac{120{,}000}{70\% \text{ of } 200{,}000} \cdot 175{,}000 = 150{,}000.$

But the payment will be limited to the policy limit of 120,000.

2.8

Claim	250	750	1000
Deductible	250	X	0

The deductible is $X = \frac{2}{3}(0) + \frac{1}{3}(250) = 83.33$, so the payment is $750 - 83.33 = 666.67$.

2.9 (a) Contributory negligence: It used to be that if a worker contributed in any way to the injury or sickness, then the worker could not seek compensation.

(b) Fellow-servant: If a fellow worker contributed in any way to the worker's injury or sickness, then the employer was not at fault and the worker could not seek compensation.

(c) Assumption-of-risk: The ability of the worker to sue was often restricted if the worker had advance knowledge of the inherent dangers of the job.

2.10 Objectives of workers compensation:

1. Broad coverage of workers for occupational injury and disease.
2. Substantial protection against loss of income.
3. Sufficient medical care and rehab services.
4. Encouragement of safety (through lower premiums).
5. An efficient and effective administrative system.

2.11 Workers compensation benefits:

1. Medical care benefits (normally unlimited).
2. Disability income benefits (after waiting period).
3. Death benefits including a burial allowance plus cash-income payments to eligible survivors.
4. Rehab services and benefits.

2.12 Advantages:

1. Gets rid of small claims and their expenses.
2. All losses are reduced by amount of deductible, so premium is lower.
3. Provides an economic incentive for the policyholder to prevent a claim.
4. Policyholders can optimize the use of their limited premium dollars by using deductibles to save money where the utility value of the coverage is not as great.

Disadvantages:

1. Insured may be disappointed by being put at risk.
2. Can lead to misunderstandings and bad public relations.
3. Makes the marketing of the coverage more difficult.
4. The insured may just inflate the claim to recover the deductible which, in turn, penalizes the honest policyholder because of the resulting higher premium.

2.13 (a) Pay (.80)(12,000) = 9600, which is within the policy limit.

(b) Pay $(12{,}000 - 1{,}000) = 11{,}000$; but the payment is limited to the policy limit of 10,000.

(c)

Claim	5000	12,000	15,000
Deductible	5000	X	0

The deductible is $X = \left(\frac{3}{10}\right)(5000) + \left(\frac{7}{10}\right)(0) = 1{,}500$;

the payment is $12{,}000 - 1{,}500 = 10{,}500$.

2.14 Let L denote the loss. If $L < d$, the claim payment is 0, and if $L > d$, the claim payment is $L - d$.

$$\begin{aligned} E[L] &= \frac{1}{10}\left[\int_0^d 0\, dL + \int_d^{10} (L-d)\, dL\right] \\ &= \frac{1}{10}\left[\frac{1}{2}L^2 - dL\Big|_d^{10}\right] = \frac{1}{10}\left[50 - 10d + \frac{1}{2}d^2\right] \end{aligned}$$

Since this must equal 2, we find d from

$$2 = \frac{1}{10}\left[50 - 10d + \frac{1}{2}d^2\right],$$

which solves for $d = 3.68$.

2.15 250 Deductible:

$$\begin{aligned} E[L] &= \frac{1}{5000}\left[\int_0^{250} 0\, dL + \int_{250}^{5000} (L-250)\, dL\right] \\ &= \frac{1}{5000}\left[\frac{1}{2}L^2 - 250L\Big|_{250}^{5000}\right] = 2256.25 \end{aligned}$$

500 Deductible:

$$\begin{aligned} E[L] &= \frac{1}{5000}\left[\int_0^{500} 0\, dL + \int_{500}^{5000} (L-500)\, dL\right] \\ &= \frac{1}{5000}\left[\frac{1}{2}L^2 - 500L\Big|_{500}^{5000}\right] = 2025.00 \end{aligned}$$

The expected loss payment will be reduced by $2256.25 - 2025.00 = 231.25$.

2.16 Reasons for policy limits:

1. Clarifies obligation of insurer.
2. Provides an upper bound on risk to insurer, decreasing the probability of insurer insolvency. Also decreases the premium.
3. Makes sure that policyholder cannot profit from a loss.
4. Allows the policyholder to choose the most appropriate coverage at an appropriate price.

Chapter 3

Note: Some exercises in this chapter were done on a pocket calculator. Students are encouraged to use Excel, but the answers may be slightly different than the ones presented here (due to rounding).

3.1 The gross IBNR is the pure IBNR, plus the development in known claims, plus the files that are closed but may reopen.

3.2 It would be better to use incurred loss development. The increase in the retention limit will be reflected immediately in the incurred data, but will not be reflected for quite some time in the paid data.

3.3 Salvage: The insurance company receives any salvage value of, for example, a car for which full payment has been made to the insured.

Subrogation: The insurance company acquires the policyholder's right to sue a third party and has this right to recover monies.

Both salvage and subrogation will reduce loss development factors, and may make them less than one.

3.4 The loss reserve would be developed in the following two steps:

1. Expected Ultimate Losses
   = (Actual Earned Premiums) · (Expected Loss Ratio)
2. Loss Reserve = (Expected Ultimate Losses) − (Paid-to-Date)

3.5 For all three models the case reserve is

$$(419.7-231.3)+(465.5-301.7)+(456.7-349.8)+(442.2-380.2) = 521.1$$

(i.e., the cumulative incurred minus the paid-to-date). From the text data we first find the loss development factors (for each model), and then the total loss reserves (see tables on next page). Finally, the gross IBNR is the excess of the total loss reserve over the case reserve.

Ratio of Successive Development Years				
Accident Year	1/0	2/1	3/2	4/3
AY1	1.250	1.100	1.082	1.008
AY2	1.250	1.085	1.088	1.016
AY3	1.225	1.093	1.094	1.024
AY4	1.232	1.100	1.100	
AY5	1.240	1.107		
AY6	1.246			
Average:	1.240	1.097	1.091	1.016
4-Year:	1.236	1.096	1.091	1.016
Mean:	1.240	1.098	1.092	1.017

Development Table (Average)					
Accident Year	1	2	3	4	Reserve for Year
AY4				449.4	69.2
AY5			498.3	506.5	156.7
AY6		510.7	557.2	566.3	264.6
AY7	520.6	571.1	623.1	633.3	402.0
			Total Loss Reserve		892.5

Development Table (4-Year Average)					
Accident Year	1	2	3	4	Reserve for Year
AY4				449.4	69.2
AY5			498.3	506.5	156.7
AY6		510.3	556.8	565.9	264.2
AY7	518.4	568.5	620.3	630.5	399.2
			Total Loss Reserve		889.3

Development Table (Mean)					
Accident Year	1	2	3	4	Reserve for Year
AY4				449.8	69.6
AY5			498.7	507.3	157.5
AY6		511.0	558.1	567.6	265.9
AY7	520.4	571.3	623.9	634.6	403.3
			Total Loss Reserve		896.2

Calculations carried to more decimal places than those shown.

3.6 Given the data in the text, we can construct the following tables.

Ratio of Successive Development Years

Accident Year	1/0	2/1	3/2	4/3	5/4	6/5
AY1	1.502	1.075	1.077	1.020	1.003	1.000
AY2	1.308	1.074	1.077	1.054	1.011	
AY3	1.366	1.155	1.117	1.044		
AY4	1.580	1.285	1.128			
AY5	1.596	1.146				
AY6	1.812					
Average:	1.527	1.147	1.100	1.039	1.007	1.000
5-Year:	1.532	1.147	1.100	1.039	1.007	1.000
Mean:	1.519	1.153	1.104	1.042	1.008	1.000

(a) Development Table (Average)

Accident Year	1	2	3	4	5	6	Reserve for Year
AY2						5199	0
AY3					6766	6766	46
AY4				6477	6521	6521	290
AY5			5881	6113	6155	6155	806
AY6		6214	6832	7102	7120	7150	1733
AY7	5982	6862	7545	7843	7896	7896	3979
		Total Loss Reserve					6853

(b) Development Table (5-Year Average)

Accident Year	1	2	3	4	5	6	Reserve for Year
AY2						5199	0
AY3					6766	6766	46
AY4				6477	6521	6521	290
AY5			5881	6113	6155	6155	806
AY6		6214	6832	7102	7150	7150	1733
AY7	6002	6885	7570	7869	7922	7922	4005
		Total Loss Reserve					6879

(c) Development Table (Mean)

Accident Year	1	2	3	4	5	6	Reserve for Year
AY2						5199	0
AY3					6772	6772	52
AY4				6495	6545	6545	314
AY5			5906	6156	6204	6204	855
AY6		6247	6898	7190	7246	7246	1829
AY7	5949	6861	7575	7896	7957	7957	4040
		Total Loss Reserve					7089

Calculations carried to more decimal places than those shown.

3.7

Year	Earned Premium ($,000)	Expected Loss Ratio	$E[\$Loss] = E[LR] \times E.P.$	f	$EL\left(1-\frac{1}{f}\right)$
AY4	4,750	0.60	2850.0	1	0
AY5	5,175	0.62	3208.5	1	0
AY6	5,500	0.65	3575.0	1.029494	102.4186
AY7	5,900	0.63	3717.0	1.329774	921.7891
				Reserve:	1024.208

3.8 Claims to be paid in the future

$$= \text{Ultimate Total}\left(1-\frac{1}{f_{ult}}\right)$$

$$= .65 \times 1000 \times \left(1-\frac{1}{1.21}\right) = 112.81$$

3.9 (a) Expected Loss Ratio Method

Accident Year	$E[LR]$	E[$Loss]	Paid-to-Date	Reserve
AY4	.680	17,000	17,000	0
AY5	.688	20,468	20,475*	0
AY6	.700	23,100	21,750	1,350
AY7	.700	26,600	17,475	9,125
			TOTAL	10,475

$^{*}20{,}475 = 12{,}050 + 6{,}025 + 2{,}400$

(b) Chain Ladder Method

Accident Year	Cumulative Claims Paid 0	1	2	3
AY4	10,000	15,000	17,000	17,000
AY5	12,050	18,075	20,475	
AY6	14,500	21,750		
AY7	17,475			

Accident Year	Loss Development Factors 1/0	2/1	3/2
AY4	1.500	1.133	1.000
AY5	1.500	1.133	
AY6	1.500		

Accident Year	Chain Ladder Development				Paid-to-Date	Reserve for Year
	0	1	2	3		
AY5				20,475	20,475	0
AY6			24,642	24,642	21,750	2,892
AY7		26,212	29,699	29,699	17,475	12,224
					Total Loss Reserve	15,116

(c) Bornhuetter-Ferguson.

Accident Year	$E[LR]$	$E[\$Loss]$	f_{ult}	B.F. Reserve
AY4	.680	17,000	1.000	0
AY5	.688	20,468	1.000	0
AY6	.700	23,100	1.133	2,712
AY7	.700	26,600	1.500×1.133	10,948
			Total	13,660

3.10 First we find the expected number of claims in each accident year using a normal chain ladder approach.

Accident Year	Loss Development Factors					Ultimate Year
	1/0	2/1	3/2	4/3	5/4	
AY2	1.125	1.055	1.042	1.010	1.000	6203
AY3	1.125	1.055	1.042	1.010		6372
AY4	1.125	1.055	1.042			6505
AY5	1.125	1.055				6512
AY6	1.125					7523
AY7	—					8250
Average	1.125	1.055	1.042	1.010	1.000	

Now the payment pattern of Exercise 3.13 can be presented in the form of payments per claim incurred (either cumulative or non-cumulative). For example, one common approach used in industry is to divide each entry of the cumulative payment triangle by the estimated ultimate number of claims by accident year. This produces the following average severity triangle.

Accident Year	Payment per Claim Incurred at Development Year t					
	0	1	2	3	4	5
AY2	30.95	71.42	96.08	119.46	135.26	135.26
AY3	32.17	76.11	106.72	130.26	146.26	
AY4	35.36	88.41	123.77	156.37		
AY5	44.24	107.22	149.46			
AY6	52.91	127.76				
AY7	64.26					

From this point we can develop this triangle of payments per claim incurred so as to ascertain an estimate of the ultimate payment per claim incurred (PPCI) for each accident year. Then for each accident year:

$$E[\text{Dollars of Ultimate Claims}] = E[PPCI] \cdot E[\text{Ultimate Number of Claims}]$$

Working with the severity triangle often proves to be more manageable and accurate than working with total claims dollars. Using the average loss development factors we obtain the following payment per claim incurred development factors.

Accident Year	Payment per Claim Incurred Development Factors				
	1/0	2/1	3/2	4/3	5/4
AY2	2.303	1.346*	1.243	1.132	1.000
AY3	2.363	1.402	1.221	1.123	
AY4	2.497	1.400	1.263		
AY5	2.425	1.394			
AY6	2.414				
Average	2.400	1.386	1.242	1.128	1.000

$^*1.346 = 96.1/71.4$

Accident Year	Future Cumulative Payments per Claim				
	1	2	3	4	5
AY4				176.3	176.3
AY5			185.6	209.3	209.3
AY6		177.0	219.8*	248.0	248.0
AY7	154.1	213.6	265.2	299.2	299.2

$^*219.8 = 127.7 \times 1.386 \times 1.242$

Accident Year	Paid-to-Date	Ultimate Paid	Liability per Claim	No. of Claims	Reserve for Year
AY2	135.3	135.3	0.0	6203	0.0
AY3	146.3	146.3	0.0	6372	0.0
AY4	156.3	176.3	20.0	6505	130.1
AY5	149.4	209.3	59.9	6512	390.1
AY6	127.7	248.0	120.3	7523	905.0
AY7	64.2	299.2	235.0	8250	1938.8
			Total Loss Reserve		3364.0

3.11

Accident Year	Incremental Severity (000) through Development Year				
	0	1	2	3	4
AY4	5.00	13.33	20.00	27.50	40.00
AY5	5.42	13.68	19.43	33.43	
AY6	4.76*	14.62	22.67		
AY7	5.47	16.00			
AY8	6.33				

$* \, 4.76 = \frac{2380}{500}$

Accident Year	Incremental Severity (000) at AY8 Level through Development Year				
	0	1	2	3	4
AY4	6.08	16.21	24.31	33.43	48.62
AY5	6.27	15.83	22.49	38.70	
AY6	5.25*	16.12	24.99		
AY7	5.75	16.80			
AY8	6.33				
Average	5.94	16.24	23.93	36.06	48.62

$* \, 5.25 = 4.76 \times 1.05^2$

Accident Year	Incremental Severity (000) through Development Year				
	0	1	2	3	4
AY5					42.00
AY6				32.71*	44.10
AY7			22.79	34.34	46.31
AY8		16.24	23.93	36.06	48.62

$* \, 32.71 = 36.06 \times 1.05^{-2}$

Accident Year	Percentage Closed Claims through Development Year				
	0	1	2	3	4
AY4	40.0%	50.0%	50.0%	53.3%	100.0%
AY5	40.0%	43.1%	51.2%	70.0%	
AY6	35.7%	50.0%	53.3%		
AY7	38.0%	51.6%*			
AY8	40.0%				
Average	38.7%	48.7%	51.5%	61.7%	100.0%

$$* \ 51.6\% = \frac{480}{1500 - 570}$$

Accident Year	Incremental Close Claims (000) through Development Year				
	0	1	2	3	4
AY4	400	300	150	80	70
AY5	480	310	210	140	*60*
AY6	500	450	240	*130*	*80*
AY7	570	480	*232**	*134*	*84*
AY8	600	*438*	*238*	*138*	*86*

$$* \ 232 = (1500 - 570 - 480) \times 0.515$$

Accident Year	Incremental Paid Losses (000) through Development Year				
	0	1	2	3	4
AY5					2,520
AY6				4,252	3,528
AY7			5,287	4,602*	3,890
AY8		7,113	5,695	4,976	4,181
Loss Reserve					46,045

$$* \ 4602 = 134 \times 34.34$$

3.12 Using the data from Exercise 3.6, we can develop the following payment patterns, first undiscounted and then discounted.

(a)

Accident Year	Average Factor Model 0	1	2	3	4	5	6
AY2							0
AY3						46	0
AY4					246	44	0
AY5				532	232	41	0
AY6			797	618	270	48	0
AY7		2065	880	683	298	53	0

Accident Year	0	1	2	3	4	5	6	Reserve for Year
AY2							0	0
AY3						44	0	44
AY4					238	40	0	278
AY5				514	210	35	0	759
AY6			770	558	228	38	0	1594
AY7		1996	795	577	235	39	0	3642
					Total Loss Reserve			6317

(b)

Accident Year	5-Year Average Factor Model 0	1	2	3	4	5	6
AY2							0
AY3						46	0
AY4					246	44	0
AY5				532	232	41	0
AY6			797	618	270	48	0
AY7		2085	883	685	299	53	0

Accident Year	0	1	2	3	4	5	6	Reserve for Year
AY2							0	0
AY3						44	0	44
AY4					238	40	0	278
AY5				514	210	35	0	759
AY6			770	558	228	38	0	1594
AY7		2016	798	578	236	39	0	3642
					Total Loss Reserve			6342

(c)

Accident Year	Mean Factor Model						
	0	1	2	3	4	5	6
AY2							0
AY3						52	0
AY4					264	50	0
AY5				557	250	47	0
AY6			830	651	292	55	0
AY7		2032	912	715	321	61	0

Accident Year	0	1	2	3	4	5	6	Reserve for Year
AY2							0	0
AY3						50	0	50
AY4					255	45	0	300
AY5				538	226	40	0	804
AY6			802	588	247	43	0	1680
AY7		1964	824	604	253	45	0	3690
					Total Loss Reserve			6524

Entries have been rounded to the nearest dollar.

3.13 (a) Given the data in the text, we can construct the following tables:

Accident Year	Cumulative Payments (in thousands)					
	0	1	2	3	4	5
AY2	192	443	596	741	839	839
AY3	205	485	680	830	932	
AY4	230	575	805	1017		
AY5	288	698	973			
AY6	398	961				
AY7	530					

Accident Year	Loss Development Factors				
	1/0	2/1	3/2	4/3	5/4
AY2	2.307	1.345	1.243	1.132	1.000
AY3	2.366	1.402	1.221	1.123	
AY4	2.500	1.400	1.263		
AY5	2.424	1.394			
AY6	2.415				
Average:	2.402	1.385	1.242	1.128	1.000
Mean:	2.408	1.388	1.244	1.127	1.000

Accident Year	Payments by Development Year (Average)					Reserve for Year
	1	2	3	4	5	
AY3					932.0	0.0
AY4				1146.7	1146.7	129.7
AY5			1208.9	1363.1	1363.1	390.1
AY6		1331.3	1654.1	1865.1	1865.1	904.1
AY7	1273.2	1763.8	2191.4	2471.0	2471.0	1941.0
			Total Loss Reserve			3364.9

Accident Year	Payments by Development Year (Mean)					Reserve for Year
	1	2	3	4	5	
AY3					932.0	0.0
AY4				1146.5	1146.5	129.5
AY5			1210.1	1364.1	1364.1	391.1
AY6		1333.4	1658.3	1869.4	1869.4	908.4
AY7	1276.4	1771.0	2202.5	2482.9	2482.9	1952.9
			Total Loss Reserve			3381.9

(b) From the data in part (a), we can construct the following tables:

Accident Year	Incremental Payments by Development Year (Average)				
	1	2	3	4	5
AY3					0.0
AY4				129.7	0.0
AY5			235.9	154.2	0.0
AY6		370.3	322.7	211.0	0.0
AY7	743.2	490.6	427.6	279.6	0.0

Accident Year	Discounted Payments by Development Year (Average)				Reserve for Year
	1	2	3	4	
AY3					
AY4				126.6	126.6
AY5			230.2	143.3	373.5
AY6		361.4	300.0	186.8	848.1
AY7	725.3	456.0	378.5	235.7	1795.4
			Total Discounted Loss Reserve		3143.7

Accident Year	Incremental Payments by Development Year (Mean) 1	2	3	4
AY4				129.5
AY5			237.1	154.0
AY6		372.4	324.9	211.1
AY7	746.4	494.7	431.5	280.4

Accident Year	Discounted Payments by Development Year (Mean) 1	2	3	4	Reserve for Year
AY4				126.4	126.4
AY5			231.3	143.2	374.5
AY6		363.5	301.9	186.9	852.3
AY7	728.4	459.7	381.9	236.4	1806.4
		Total Discounted Loss Reserve			3159.6

Calculations were carried to more decimals than those shown.

3.14

Months	Incremental Percentage Paid	Discount Factor	Product
12	20	—	
24	25*	$(1.04)^{-1/2}$	24.51452
36	15	$(1.04)^{-3/2}$	14.14299
48	15	$(1.04)^{-5/2}$	13.59903
60	15	$(1.04)^{-7/2}$	13.07599
72	10	$(1.04)^{-9/2}$	8.38204
			73.71457

* 25 = 45−20

Then the discounted loss reserve is

$$16{,}000{,}000 \times \frac{73.71457}{100} = 11{,}794{,}331.$$

3.15 The mean loss development factors are 1.51873, 1.15325, 1.10420, 1.04233, 1.00771, and 1.00000. First we determine the cumulative loss payments through development year 6 for all accident years using these loss development factors, and the resulting incremental loss payments (i.e. loss reserves).

Accident	Cumulative Loss Payments through Development Years						
Year	0	1	2	3	4	5	6
AY1							3166.00
AY2						5199.00	5199.00
AY3					6720.00	6771.81	6771.81
AY4				6231.00	6494.76	6544.83	6544.83
AY5			5349.00	5906.37	6156.39	6203.86	6203.86
AY6		5417.00	6247.16	6898.11	7190.11	7245.55	7245.55
AY7	3917.00	5948.87*	6860.53	7575.40	7896.07	7956.95	7956.95

* $5948.87 = 3917 \times 1.51873$

Accident	Estimated Mean Loss Reserves through Development Years						
Year	1	2	3	4	5	6	Total
AY1							
AY2						0.00	0.00
AY3					51.81	0.00	51.81
AY4				263.76	50.07	0.00	313.83
AY5			557.37	250.02	47.47	0.00	854.86
AY6		830.16	650.95	292.00	55.44	0.00	1828.55
AY7	2031.87	911.66*	714.87	320.67	60.88	0.00	4039.95

* $911.66 = 6860.53 - 5948.87$

The allocated loss reserves are determined by adjusting each amount by the ratio of the adopted loss reserve to the mean loss reserve.

Also shown are the factors used to discount the allocated loss reserves and the discounted loss reserves.

Accident Year	Allocated Loss Reserves through Development Years 1	2	3	4	5	6	Total
AY1						0	0
AY2						0	0
AY3					48.00	0	48.00
AY4				250.46	47.54	0	298.00
AY5			535.95	240.41	45.65	0	822.00
AY6		801.31	628.33	281.85	53.51	0	1765.00
AY7	2015.80	904.45*	709.22	318.13	60.40	0	4008.00
Discount Factors	0.96900	0.90986	0.85433	0.80219	0.75323		

$$* \; 904.45 = 911.66 \times \frac{4008.00}{4039.95}$$

Accident Year	Discounted Allocated Loss Reserves through Development Years 1	2	3	4	5	6	Total
AY1						0	0
AY2						0	0
AY3					46.51	0	46.51
AY4				242.69	43.26	0	285.95
AY5			519.33	218.74	39.00	0	777.07
AY6		776.47	571.69	240.79	42.93	0	1631.88
AY7	1953.32	822.92	605.91	255.20	45.49	0	3682.85
							6424.26

3.16 If future cash flows are negative (e.g., because of salvage or subrogation) then discounted loss reserves could exceed undiscounted loss reserve.

3.17 (a) We are given the incremental severity at AY5 level through development year, so we proceed by completing the bottom half of the incremental severity triangle.

Accident Year	Incremental Severity (000) through Development Year				
	0	1	2	3	4
AY2					13,468.91
AY3				9,782.51*	14,411.74
AY4			8,785.05	10,467.29	15,420.56
AY5		7,300.00	9,400.00	11,200.00	16,500.00

$^{*}9{,}782.51 = 11{,}200 \times 1.07^{-2}$

Accident Year	Incremental Close Claims (000) through Development Year				
	0	1	2	3	4
AY1	310	190	155	140	45
AY2	330	205	170	155	*50*
AY3	360	215	185	*161*	*54*
AY4	350	220	*182**	*160*	*53*
AY5	410	*256*	*209*	*184*	*61*

$^{*}182 = (965 - 350 - 220) \times 0.46$

Accident Year	Incremental Paid Losses (000) through Development Year				
	0	1	2	3	4
AY2					673,446
AY3				1,574,984	778,234
AY4			1,598,879	1,674,766*	817,290
AY5		1,868,800	1,964,600	2,060,800	1,006,500
Loss Reserve					14,018,299

$^{*}1{,}674{,}766 = 10{,}467.29 \times 160$

(b) Discounted loss reserve:

Accident Year	Incremental Paid Losses (000) through Development Year				
	0	1	2	3	4
AY2					660,368
AY3				1,544,399	733,770
AY4			1,567,830	1,579,080*	740,956
AY5		1,832,509	1,852,355	1,868,325	877,399
Loss Reserve					13,256,992

$^*1{,}579{,}080 = 1{,}674{,}766 \times 1.04^{-1.5}$

3.18 (a) Incurred triangle will include case reserve estimates and paid-to-date. Paid triangle is purely paid-to-date.
Ultimately, incurred = paid.

(b) Yes, for three reasons:
(i) overly conservative case reserve estimates,
(ii) salvage,
(iii) subrogation.

(c) LDF should normally be larger for a paid triangle.

(d)

Accident Year	Incurred Losses by DY			
	0	1	2	3
AY4	2,147	2,202	2,214	2,214
AY5	2,312	2,390	2,402	
AY6	2,451	2,520		
AY7	2,612			

Accident Year	LDFs		
	1/0	2/1	3/2
AY4	1.02562	1.00545	1.00000
AY5	1.03374	1.00502	
AY6	1.02815		
AY7			
Average	1.02917	1.00524	1

Accident Year	Undiscounted Incremental Incurred Losses by DY 1	2	Total
AY4			
AY5			0
AY6		13.1928	13.1928
AY7	76.1885	14.0734	90.2619
	Total Undiscounted Liability		103.4547

(e)

Accident Year	Discounted Incremental Incurred Losses by DY 1	2	Total
AY4			
AY5			0
AY6		12.7540	12.7540
AY7	73.6542	12.7152*	86.3694
	Total Discounted Liability		99.1234

Liability (assuming payments made mid-year)
$^*12.7152 = 14.0734 \times 1.07^{-1.5}$

3.19 Look at your loss development factors

Accident Year	1/0	2/1	3/2	4/3
AY3	1.200	1.083	1.03846	1.000
AY4	1.424	1.085	1.03922	
AY5	1.195	1.082		
AY6	1.205			
	1.200*	1.083	1.03900	1.000

*average dropping 1.424

(a) Clearly the "1.424" link ratio is wrong. It would appear that data point "165" is in error. However, we can drop it from the distribution without harm.

(b) Using "mean" LDF,
Reserve = 110.87 as follows

LDF	1/0	2/1	3/2	4/3
Mean	1.248	1.083	1.039	1.000

Accident Year	Cumulative Paid Losses					Paid-to-date	Reserve
	0	1	2	3	4		
AY3	200.00	240.00	260.00	270.00	270.00	270.00	0
AY4	165.00	235.00	255.00	265.00	265.00	265.00	0
AY5	205.00	245.00	265.00	275.34	275.34	265.00	10.34
AY6	195.00	235.00	254.51	264.43	264.43	235.00	29.43
AY7	203.00	243.60	263.82	274.11	274.11	203.00	71.11
						Total	110.87

(c) Cash flow in future using "mean" LDF

Accident Year	Year of Payment		
	AY8	AY9	AY10
AY5	10.34		
AY6	19.51	9.93	
AY7	40.60	20.22	10.29
	70.44	30.14	10.29
	$*v^{1/2}$	$*v^{1.5}$	$*v^{2.5}$

$$= 68.74 + 28.02 + 9.11 = \$105.87$$

Assuming all payments made mid-year.

3.20 Accident Year AY3 is at 60 months at December 31, AY7. So for Accident Year AY3, there is no reserve liability.

Now,

AY4:

$$E[ULT(\$L)] = 1{,}000{,}000(1.05)(1.03) = 1{,}081{,}500$$

Still to be paid = 5% of 1,081,500 = 54,075

AY5:

$$E[ULT(\$L)] = 1{,}000{,}000(1.05)^2(1.03)^2 = 1{,}169{,}642$$

Still to be paid = 15% of 1,169,642 = 175,446

AY6:

$$E[ULT(\$L)] = 1{,}000{,}000(1.05)^3(1.03)^3 = 1{,}264{,}968$$

Still to be paid $= 30\%$ of $1{,}264{,}968 = 379{,}490$

AY7:

$$E[ULT(\$L)] = 1{,}000{,}000(1.05)^4(1.03)^4 = 1{,}368{,}063$$

Still to be paid $= 60\%$ of $1{,}368{,}063 = 820{,}838$

Total Reserve = \$1,429,849

Chapter 4

4.1 (a) Calendar Year CY4

Occurrence #1	1,000
Occurrence #2	0
Occurrence #3	0
	1,000

(b) Calendar Year CY6

Occurrence #1	1,000
Occurrence #2	5,000
Occurrence #3	5,000
	11,000

(c) Accident Year AY5 (as of 12/31/CY6)

Occurrence #1	0
Occurrence #2	5,000
Occurrence #3	5,000
	10,000

4.2 Only two events contribute to the answer:

Claim #2:	200 (Paid in CY5) + 200(12/31/CY6 Reserve)	=	400
Claim #3:	300 (Paid in CY6) + 0(12/31/CY6 Reserve)	=	300
	Total		700

4.3 $\dfrac{\text{Incurred Losses}}{\text{Earned Premiums}}$

$$= \frac{\text{Paid Losses} + (\text{CY6 Reserve} - \text{CY5 Reserve})}{\text{Written Premium} + (\text{CY5 Unearned} - \text{CY6 Unearned})}$$

$$= \frac{90{,}000 + (140{,}000 - 160{,}000)}{100{,}000 + (50{,}000 - 40{,}000)} = .636$$

4.4 One-half of six-month written premium will be fully earned, and the rest will be half earned. For one-year policies, written premium will be half earned by year-end. Thus we have

Unearned premium

$$= \frac{1}{2}\left[\frac{1}{2}(24{,}000{,}000) + (120{,}000{,}000)\right] = 66{,}000{,}000$$

But

$$\text{Dollars earned} = 144{,}000{,}000 - 66{,}000{,}000 = 78{,}000{,}000$$

4.5 State A has a uniform distribution of renewals. Assuming a uniform distribution of claims, the average accident incurral date is January 1, CY8.

State B has all policies renewing on January 1, CY7. Assuming a uniform distribution of claims, the average accident incurral date is July 1, CY7.

4.6 The mid-point of the CY5 accident year is July 1, CY5, and the mid-point of the future exposure period is August 1, CY7. This is a distance of 2 years and 1 month, so $t = 2\frac{1}{12}$ years.

4.7 This could result from one or more of the following, which is not intended to be a complete list.

1. Trend factor greater than zero.
2. A change in the permissible loss ratio, and hence in the expense ratio.
3. A change in the externalities (e.g., speed limit), where the effect is not yet felt.

4.8 The claim is not true. The trend in loss cost (i.e., trend factor) is from average date of incurral in past historical period to average date of incurral in future exposure period.

Inflation in the loss development factor is to cover the period from average date of incurral in future exposure period to average date of settlement.

4.9 The mid-point of policy year CY4 is January 1, CY5, the mid-point of policy year CY5 is January 1, CY6, and the experience period mid-point is November 1, CY8. Thus the CY4 loss cost will be projected 3 years, 10 months (23/6 years) and the CY5 loss cost will be projected 2 years, 10 months (17/6 years). The weighted projected loss cost is

$$(.40)(200)e^{(.10)(23/6)} + (.60)(217)e^{(.10)(17/6)} = 290.22.$$

4.10 Using formula (4.3), Gross Rate $= 600 = \dfrac{\text{Loss Cost} + 75}{1-.14-.05}$

Solves for, Loss Cost = 411.

With 10% increase in Loss Cost, new gross rate is:

Gross Rate $= \dfrac{411 \times 1.1 + 75}{1-.14-.05} = 650.74.$

4.11 Given Gross Premium: 500

Old Loss Cost: .67(500) = 335

New Loss Cost: 335

New Commission: 50

All Other Expenses: .21

Then the new average gross premium will be $\dfrac{335}{.79} + 50 = 474.05$.

4.12 The current gross rate is 1000, leading to the following expense table.

	Old Basis	New Basis
Loss Cost	640	640
Commission	20%	12%
General Expenses	80	80
Taxes	3%	3%
Profit	50	50

The loss cost, general expenses, and profit amount remain the same, but they now represent 85% of the gross premium. Therefore the new gross premium is $\frac{640+80+50}{.85} = 905.88$.

4.13 For Company A, the gross premium is $\frac{LC}{.700}$, where LC denotes the loss cost.

For Company B, the gross premium is $\frac{LC(1+i)^{-1/2}}{1-.325}$, since expenses other than profit and contingencies represent 27.5% and profit and contingencies represents 5%, a total of 32.5%. Equating and solving for i we have

$$\frac{LC}{.700} = \frac{LC(1+i)^{-1/2}}{.675},$$

from which we find $i = \left(\frac{.700}{.675}\right)^2 - 1 = 7.54\%$.

4.14 (a) $Z = \sqrt{\frac{E}{K}}$, for $E \leq K$

$Z = 1$ for $E > K$

Properties:

1. $0 \leq Z \leq 1$? Yes

2. $\frac{dZ}{dE} > 0$?

$$\frac{dZ}{dE} = \frac{d}{dE}\left(\frac{E}{K}\right)^{1/2} = \frac{1}{2}\left(\frac{E}{K}\right)^{-1/2} > 0$$ Yes

3. $\frac{d}{dE}\left(\frac{Z}{E}\right) < 0?$

$$\frac{d}{dE}\left(\frac{Z}{E}\right) = \frac{d}{dE}(EK)^{-1/2} = -\frac{1}{2}(EK)^{-3/2} < 0$$ Yes

(b) $Z = \frac{E}{E+K}$

Properties:

1. $0 \le Z \le 1?$ Yes

2. $\frac{dZ}{dE} > 0?$

$$\frac{dZ}{dE} = \frac{(E+K)-E}{(E+K)^2} = \frac{K}{(E+K)^2} > 0$$ Yes

3. $\frac{d}{dE}\left(\frac{Z}{E}\right) < 0?$

$$\frac{d}{dE}\left(\frac{Z}{E}\right) = \frac{d}{dE}(E+K)^{-1} = -(E+K)^{-2} < 0$$ Yes

4.15 (a) The derivative of the curve $Z = \frac{E}{E+K}$ at $E = Q$ has the same slope as the tangent line from the point $\left(Q, \frac{Q}{Q+K}\right)$ to the point $(S,1)$. The slope of the curve is $\frac{d}{dE}\left(\frac{E}{E+K}\right) = \frac{K}{(E+K)^2}$, which is $\frac{K}{(Q+K)^2}$ at $E = Q$.

The slope of the tangent line is $\frac{1-\frac{Q}{Q+K}}{S-Q} = \frac{K}{(Q+K)(S-Q)}$.

Equating we have $\frac{K}{(Q+K)^2}=\frac{K}{(Q+K)(S-Q)}$, from which we find $Q+K = S-Q$, so that $Q=\frac{1}{2}(S-K)$.

(b) For $Q \leq E \leq S$, the function follows the tangent line with slope $\frac{K}{(Q+K)(S-Q)} = \frac{K}{(Q+K)^2}$, since $(Q+K) = (S-Q)$.

Using a point-slope formula with $Z=1$ at $E=S$, we have

$$\frac{1-Z}{S-E} = \frac{K}{(Q+K)(S-Q)} = \frac{K}{(Q+K)^2},$$

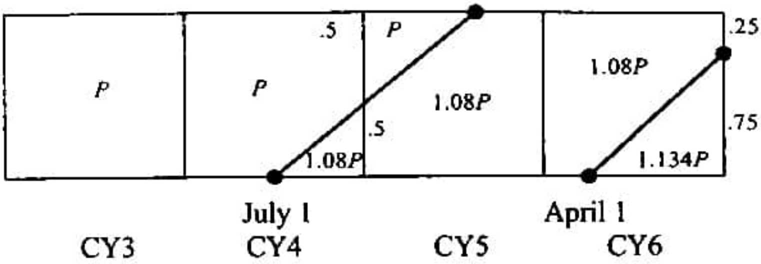
where $Q=\frac{1}{2}(S-K)$. Substituting for Q we have

$$\frac{1-Z}{S-E} = \frac{4K}{(S+K)^2}.$$

Then

$$\begin{aligned} Z &= 1-\frac{(S-E)(4K)}{(S+K)^2} \\ &= \frac{S^2+2KS+K^2-4KS+4KE}{(S+K)^2} \\ &= \frac{(S^2-2KS+K^2)+4KE}{(S+K)^2} = \frac{(S-K)^2+4KE}{(S+K)^2}. \end{aligned}$$

4.16



CY4: 1/8 at $1.08P$ and 7/8 at P, producing an Average Earned Premium of $1.01P$. We want $1.134P$, so the on-level factor is 1.134/1.01. Thus, CY4 Earned Premiums at Current Rate Levels $= 1.134/1.01 \times 2927 = 3286.35$.

CY5: 1/8 at P and 7/8 at $1.08P$, producing an Average Earned Premium of $1.07P$. We want $1.134P$, so the on-level factor is 1.134/1.07. Thus, CY5 Earned Premiums at Current Rate Levels $1.134/1.07 \times 3301 = 3498.44$.

CY6: 9/32 at 1.134 and 23/32 at $1.08P$, producing an Average Earned Premium of $1.095P$. We want $1.134P$, so the on-level factor is 1.134/1.095. Thus, CY6 Earned Premium at Current Rate Levels $1.134/1.095 \times 3563 = 3689.90$.

4.17 Using Class 1 as the base class, the new Class 2 differential is $1.10(.580/.500) = 1.276$.

4.18 The student has used the loss cost method. We should also try the loss ratio method with Class A as the base class. Then, from Equation (4.10),

$$(\text{New Differential})_i = (\text{Existing Differential})_i \cdot \frac{LR_i}{LR_A},$$

producing the following results:

Class	Loss Ratio Indicated Differential
A	1.000
B	1.092
C	1.122
D	1.176
E	1.404
F	1.428
G	1.560

The loss cost method is subject to distortions if there exist heterogeneous distributions of cross classifications (e.g., territory versus class). The loss ratio method automatically adjusts for such heterogeneity, so the loss ratio method is recommended.

4.19 The weighted average class differential is 1.25 (constant for all territories). The present base rates are as follows:

Territory	Present Base Rate	Old Differential
1	152.00	1.0000000
2	130.40	.8578947
3	96.00	.6315789

Next we find the new territorial differentials using the loss ratio method.

Territory	Old Differential	Loss Ratio	New Differential
1	1.0000000	.720	1.0000000
2	.8578947	.620	.7387426
3	.6315789	.770	.6754385

The overall rate change is +7%, or a factor of 1.07. The old average differential is .8836841, and the new average differential is .8567104.

The new base rate for Territory 1 is $152 \times 1.07 \times \frac{.8836841}{.8567104} = 167.76$, and the new base rate for Territory 3 is $167.76(.6754385) = 113.31$.

4.20 The new commission will be a constant 50, with effective date July 1, CY7. Assuming one-year policies, the mid-point of the exposure period is July 1, CY8, and the mid-point of the experience period is January 1, CY6.

The following values are then developed.

Trend Factor: $(1.09)^{2.5} = 1.2404128$

Dollars of Losses (Trended and Developed): 5,200,788

Loss Cost per Unit Exposure: $\frac{5{,}200{,}788}{30{,}493} = 170.55678$

Fixed expense per exposure = 50 (commission expense)

We can use Equation (4.6b) to determine the new average gross rate, with the modification that the commission expense will only be loaded for profit.

New Average Gross Rate: $\frac{170.55678}{1-.05-.03-.05} + \frac{50}{1-.05} = 248.67385$

Base Gross Rate: $\frac{248.67385}{1.25} = 198.94$

4.21 First we find the existing base rate for each territory.

Territory	Average Class Differential	Base Rate	Existing Differential
1	1.0433333	215.65496	1.000000
2	1.0333333	193.54839	0.897491
3	1.0114286	177.96610	0.825235

Next we find the new territorial differentials using the loss ratio method.

Territory	Existing Differential	Loss Ratio	New Differential
1	1.000000	.700	1.000000
2	0.897491	.660	0.846206
3	0.825235	.720	0.848814

One could easily argue that Territory 3 should be the base rate, but there would be no difference in the final answer since $Z = 1$ for all territories.

The overall rate change is +3%, or a factor of 1.03.

There is heterogeneity in the class distribution, so the balance-back factor must be determined.

Old Average Differential:

$$\{[2000(1.00)+150(1.10)+600(.90)+100(1.25)+150(2.00)]$$
$$+\ .897491[800(1.00)+150(1.10)+\cdots+50(2.00)]$$
$$+.825235[2250(1.00)+200(1.10)+\cdots+50(2.00]\}/8000=.9303056$$

New Average Differential:

$$\{[2000(1.00)\ +\ 150(1.10)+\cdots+150(2.00)]$$
$$+.846206[800(1.00)+150(1.10)+\cdots+50(2.00)]$$
$$+.848814[2250(1.00)+200(1.10)+\cdots+50(2.00)]\}/8000$$
$$=\ .9308024$$

Then the new base rate for Territory 1, Class A is

$$215.65496\times 1.03\times \frac{.9303056}{.9308024} = 222.01,$$

and the new rate for Territory 3, Class B is

$$222.01\times 1.10\times .848814=207.29\,.$$

4.22 Policy year PY3 mid-point is December 31, CY3 (or January 1, CY4), and policy year CY4 mid-point is December 31, CY4 (or January 1, CY5). The mid-point of the effective period is December 31, CY7 (or January 1, CY8).

Then the policy year PY3 expected loss ratio is

$$\frac{1{,}000{,}000(1.05)^4}{2{,}000{,}000} = .6077531,$$

and the policy year PY4 expected loss ratio is

$$\frac{2{,}000{,}000(1.05)^3}{3{,}000{,}000} = .77175.$$

Then the weighted loss ratio is

$$.30(.6077531)+.70(.77175) = .7225509,$$

and the indicated change is

$$\frac{\textit{Weighted Loss Ratio}}{\textit{Permissible Loss Ratio}} = \frac{.7225509}{.600} = 1.2042515,$$

a change of +20.4%.

4.23 We will use the PY4 policy year data. The mid-point of the experience period is January 1, CY5. The price is to be set July 1, CY6 for one-year policies, so the mid-point of the exposure period is July 1, CY7. Therefore the trend factor is 1.30.

To determine loss development factors, we first note the following incurred loss data.

	Incurred Losses as of Report Number			
Policy Year	0	1	2	3
PY1	?	660,000	693,000	693,000
PY2	800,000	880,000	924,000	
PY3	900,000	990,000		
PY4	1,000,000			

This leads to the following loss development factors:

1/0	2/1	∞/2
1.10	1.05	1.00

Using the loss development factors we next determine

Expected Losses (Trended and Developed)
$= 1{,}000{,}000(1.10)(1.05)(1.30) = 1{,}501{,}500.$

At this point we can proceed using either the loss ratio or the loss cost approach.

Loss Ratio Method:

Earned Premium at Current Rates:

$$10{,}000(80) + 8{,}000(150) = 2{,}000{,}000$$

Expected Loss Ratio: $\dfrac{1{,}501{,}500}{2{,}000{,}000} = .75075$

Permissible Loss Ratio: .700

Indicated Rate Change: $\dfrac{.75075}{.700} = 1.0725$, or +7.25%

To develop the territorial differential change, we set Territory A differential at 1.000.

	Territory A	Territory B
Existing Differential	1.000	$\frac{150}{80} = 1.875$
Earned Premium at Current Rates	800,000	1,000,000
Incurred Losses	520,000	480,000
Loss Ratio	.650	.400
Indicated Differential	1.000	1.1538462
Credibility Differential	1.000	1.2259615

Old Average Differential: $1.3\dot{8}$

New Average Differential: 1.1004273

Territory A Average Rate: $80 \times 1.0725 \times \dfrac{1.3\dot{8}}{1.1004273} = 108.29$

Territory B Average Rate: $108.29 \times 1.2259615 = 132.76$

Loss Cost Method:

Expected Losses: 1,501,500

Loss Cost per Unit Exposure: $\frac{1,501,500}{18,000} = 83.41\dot{6}$

Average Gross Premium: $\frac{83.41\dot{6}}{.700} = 119.1\dot{6}$

Territorial Differential Change:

Territory	Existing Differential	Loss Cost Per Unit	Indicated Differential	Credibility Differential
A	1.000	52	1.0000000	1.0000000
B	1.875	60	1.1538462	1.2259615

New Average Differential: 1.1004273

Then the Territory A Average Rate is $\frac{119.1\dot{6}}{1.1004273} = 108.29$, and the Territory B Average Rate is 108.29(1.2259615) = 132.76.

4.24 **Loss Cost Method:**

Trend Factor: $(1.08)^t$

Loss Development Factor: (1.10)(1.05) = 1.155

Time Factor: July 1, CY4 to February 1, CY7, which is 2 years and 7 months, or $2.58\dot{3}$ years

Expected Dollars of Losses:

$$1,570,000(1.08)^{2.58\dot{3}}(1.155) = 2,212,210$$

New Premium per Exposure Limit: $\frac{2,212,210}{18,000} = 122.90$

Average Gross Premium: $\frac{122.90}{.80} = 153.62569$

Differentials (adjusting for heterogeneity in cross distribution)

Territory	Loss Cost	Differential
1	$\dfrac{1{,}100{,}000}{10{,}000(1)+4{,}000(1.2)} = 74.324324$	1.0000000
2	$\dfrac{470{,}000}{2{,}000(1)+2{,}000(1.2)} = 106.81818$	1.4371902

Class	Loss Cost	Differential
1	$\dfrac{900{,}000}{10{,}000(1)+2{,}000(1.1)} = 73.77049$	1.0000000
2	$\dfrac{670{,}000}{4{,}000(1)+2{,}000(1.1)} = 108.06452$	1.4648746

Average Differential:

$$\frac{10{,}000(1)+4{,}000(1.465)+2{,}000(1.437)+2{,}000(1.465\times 1.437)}{18{,}000} = 1.2746936$$

Base Rate for Territory 1, Class 1: $\dfrac{153.62569}{1.2746936} = 120.5197$

Base Rate for Territory 2, Class 2:

$$120.5197\times 1.437\times 1.465 = 253.72$$

Loss Ratio Method:

Trend Factor: $(1.08)^t$

Loss Development Factor: $(1.10)(1.05) = 1.155$

Time Factor: July 1, CY4 to February 1, CY7, which is 2 years and 7 months, or $2.58\dot{3}$ years

Expected Dollars of Losses:

$$1{,}570{,}000(1.08)^{2.58\dot{3}}(1.155) = 2{,}212{,}210$$

Expected Loss Ratio:

$$\frac{2{,}212{,}210}{100(10{,}000)+120(4{,}000)+110(2{,}000)+132(2{,}000)} = 1.12638$$

Indicated Rate Change: $\dfrac{1.12638}{.80} = 1.4079747$

New Base Rate (unadjusted): 140.80

New Differentials:

Territory	Loss Ratio	Old Differential	New Differential
1	$\dfrac{1{,}100{,}000}{10{,}000(100)+4{,}000(120)} = .74324324$	1.00	1.0000000
2	$\dfrac{470{,}000}{2{,}000(110)+2{,}000(132)} = .9710743$	1.10	1.4371902

Class	Loss Ratio	Old Differential	New Differential
1	$\dfrac{900{,}000}{10{,}000(100)+2{,}000(110)} = .7377049$	1.00	1.0000000
2	$\dfrac{670{,}000}{4{,}000(120)+2{,}000(132)} = .9005376$	1.20	1.4648746

Old Average Differential:

$$\frac{10{,}000(1)+4{,}000(1.2)+2{,}000(1.1)+2{,}000(1.32)}{18{,}000} = 1.09\dot{1}$$

New Average Differential:

$$\frac{10{,}000(1)+4{,}000(1.4648746)+2{,}000(1.4371902)+2{,}000(1.4648746\times 1.4371902)}{18{,}000} = 1.2746936$$

New Base Rate (Balanced-Back):

$$140.79747\times\frac{1.091}{1.2746936} = 120.5197$$

Then the new base rate for Territory 2, Class 2 is

$$120.5197 \times 1.437 \times 1.465 = 253.72.$$

NOTE:

Earned Exposures:

	Territory 1	Territory 2
Class 1	10,000	2,000
Class 2	4,000	2,000

Old Rates:

	Territory 1	Territory 2
Class 1	100	110
Class 2	120	32

New Rates:

	Territory 1	Territory 2
Class 1	120.52	173.21
Class 2	176.55	253.73

Old Premium Income: 1,964,000

New Premium Income: 2,765,280

Ratio: 1.4079837, as required (within round-off error)

4.25 PY3 $\Rightarrow$ mid-point = Dec. 31, CY3 or Jan. 1, CY4
Trend to Jan. 1, CY8 = 4 years

$$\therefore \quad E[LR(T\&D)] = \frac{1,500,000(1.06)^4}{3,000,000} = 0.631238$$

PY4 $\Rightarrow$ mid-point = Dec. 31, CY4 or Jan. 1, CY5
Trend to Jan 1, CY8 = 3 years

$$\therefore \quad E[LR(T\&D)] = \frac{3,000,000(1.06)^3}{4,500,000} = 0.794011$$

$$\begin{aligned} Wtd\ 30|70 \Rightarrow\ & .30(.631238)+.70(.893262) \\ &= .1893714+0.555807 \\ &= .745179 \end{aligned}$$

$$\therefore \quad \text{Rate change} = \frac{.745179}{.650}-1 = +14.6\%$$

4.26 Base class is class B since $Z=1$.
This is essential.

a) Loss Ratio Method

Terr	Curr Rel	L.R.	Ind Rel	Z	Adopted Rel
A	2.000	0.600	1.760	0.900	1.784
B	1.000	0.682	1.000	1.000	1.000
C	1.818	0.725	1.934	0.500	1.876

b) To do loss cost, need exposure units.
Exposure Units = (Earned Prem) / (Base Rate)
Exposure Units = (Earned Prem) / (Base Rate)

Terr	Curr Rel	#Exp	Adj # Exp.	Loss Cost	Ind Rel	Z	Adopted Rel
A	2.000	5,500	$5{,}500\left(\frac{100}{110}\right)=5{,}000$	66.00	1.760	0.900	1.784
B	1.000	15,400	$15{,}400\left(\frac{50}{55}\right)=14{,}000$	37.50	1.000	1.000	1.000
C	1.818	4,000	$4{,}000\left(\frac{100}{100}\right)=4{,}000$	72.50	1.933	0.500	1.876

4.27 Loss Ratio

Class	Curr Rel	Loss Ratio	Ind Rel	Z	Adopted Rel
1	1.00	0.600	1.00	1.00	1.000
2	1.25	0.528	1.10	0.70	1.145
3	1.50	0.540	1.35	0.80	1.380

Loss Cost

Class	Premium	Earned Premium	No. of Exposure Units
1	100	50,000	500
2	125	20,000	160
3	150	30,000	200
			860

Class	Exist Diff	Exposure Units	L.C.	Ind. Diff	Z	Adopted Diff $Z(IND)$ $+(1-Z)(EXIST)$
1	1.000	500	\$60.00	1.000	1.000	1.000
2	1.250	160	66.00	1.100	0.700	1.145
3	1.500	200	81.00	1.350	0.800	1.380

$$\text{Old Average Rel.} = \frac{500(1)+160(1.25)+200(1.50)}{860} = 1.16279$$

$$\text{New Average Rel.} = \frac{500(1)+160(1.145)+200(1.380)}{860} = 1.11535$$

$$\therefore \quad \text{New Base Rate} = \frac{100(1.07)(1.16279)}{1.11535} = \$111.55$$

Class	New Rate
1	\$111.55
2	127.72
3	153.94

Chapter 5

5.1 Schedule rating is used initially to reflect the expected reduction in losses. Once enough history is reflected in the insured's experience, the reduction would be captured in the experience rating and be removed from the schedule rating credits to avoid double-counting.

5.2 The composite premium is based on final audited premiums. Composite premium is:

$$42.40 \times 10{,}105{,}000 \div 1{,}000 = 428{,}452$$

5.3 $Z = \dfrac{520{,}400}{500{,}000 + 520{,}400} = 0.51$

$$\text{Experience Loss Ratio} = \frac{125{,}000 + 145{,}600}{250{,}000 + 270{,}400} = 0.52$$

Experience Modification Factor

$$= \left[\left(Z \times \frac{\text{Experience Loss Ratio}}{\text{Expected Loss Ratio}} \right) + (1 - Z) \right]$$

$$= 0.51 \times \frac{0.52}{0.60} + (1 - 0.51) = 0.932$$

Experience Rated Premium

= Manual Premium × Experience Modification Factor

= 281,000×0.932 = 261,892

5.4

Size of Loss	# of Claims	Losses Capped at 200,000	Losses Capped at 1,000,000	Losses Capped at 2,000,000
1 – 200,000	384	47,001,600	47,001,600	47,001,600
200,001 – 500,000	250	50,000,000	81,050,000	81,050,000
500,001 – 1,000,000	140	28,000,000	97,076,000	97,076,000
1,000,001 – 2,000,000	53	10,600,000	53,000,000	75,737,000
2,000,001 – 5,000,000	15	3,000,000	15,000,000	30,000,000
Total	842	138,601,600	293,127,600	330,864,600

$$LAS(200,000) = \frac{138,601,600}{842} = 164,610$$

$$LAS(1,000,000) = \frac{293,127,600}{842} = 348,133$$

$$LAS(2,000,000) = \frac{330,864,600}{842} = 392,951$$

$$ILF_{1,000,000} = \frac{LAS(1,000,000)}{LAS(200,000)} = \frac{348,133}{164,610} = 2.115$$

$$ILF_{2,000,000} = \frac{LAS(2,000,000)}{LAS(200,000)} = \frac{392,951}{164,610} = 2.387$$

5.5

Size of Loss	Cum. Prob.	Incr. Prob.	Average Loss in Interval	Pure Premium
1 – 100,000	0.52	0.52	72,500	37,700
100,001 – 200,000	0.71	0.19	142,200	27,018
200,001 – 500,000	0.86	0.15	378,900	56,835
500,001 – 1,000,000	0.93	0.07	712,400	49,868
1,000,001 – 10,000,000	1.00	0.07	2,970,000	207,900

e.g. 37,700 = 0.52×72,500

(a) We can use Formula (5.6) to calculate LAS(200,000) and LAS(1,000,000).

LAS(200,000) = sum of pure premiums for each layer up to 200,000, plus 200,000 multiplied by the probability of a claim exceeding 200,000

$$= 37,700 + 27,018 + (1 - 0.71) \times 200,000 = 122,718$$

Similarly, LAS(1,000,000)

$$= 37,700 + 27,018 + 56,835 + 49,868$$

$$+ (1 - 0.93) \times 1,000,000 = 241,421$$

$$ILF_{1,000,000} = \frac{LAS(1,000,000)}{LAS(200,000)} = \frac{241,421}{122,718} = 1.97$$

(b) $$RL_{200,000} = \frac{122,718^2}{2,000,000} = 7,530$$

$$RL_{1,000,000} = \frac{241,421^2}{2,000,000} = 29,142$$

Using Formula (5.5):

$$ILF_{1,000,000} = \frac{LAS(1,000,000) + RL_{1,000,000}}{LAS(200,000) + RL_{200,000}}$$

$$= \frac{241,421 + 29,142}{122,718 + 7,530} = 2.08$$

5.6

Claim #	Ground-up Loss	Losses Eliminated at \$500 Ded.	Losses Eliminated at \$1,000 Ded.
1	400	400	400
2	5,300	500	1,000
3	10,500	500	1,000
4	15,800	500	1,000
5	23,700	500	1,000
Total	55,700	2,400	4,400

$$LER_{500} = \frac{2,400}{55,700} = 0.043$$

Indicated Deductible Relativity$_{500}$ = 1 – 0.043 = 0.957

$$LER_{1,000} = \frac{4,400}{55,700} = 0.079$$

Indicated Deductible Relativity$_{1,000}$ = 1 – 0.079 = 0.921

5.7

Size of Loss	# of Claims	Ground-Up Losses	Losses Eliminated at \$500 Ded.	Losses Eliminated at \$1,000 Ded.
1 – 500	840	285,600	285,600	285,600
501 – 1,000	1,260	945,000	630,000	945,000
1,000 – 2,000	920	1,297,200	460,000	920,000
2,001 or greater	2,180	8,611,000	1,090,000	2,180,000
Total	5,200	11,138,800	2,465,600	4,330,600

Expected Losses @ \$500 Deductible = 11,138,800 – 2,465,600
= 8,673,200

Expected Losses @ \$1,000 Deductible = 11,138,800 – 4,330,600
= 6,808,200

Using Formula (5.8):

$$\text{Indicated deductible relativity}_0 = \frac{11,138,800}{8,673,200} = 1.284$$

$$\text{Indicated deductible relativity}_{1,000} = \frac{6,808,200}{8,673,200} = 0.785$$

5.8 Total losses $= \int_0^{1,000} \frac{x^2}{500,000}dx = \frac{x^3}{1,500,000}\Big|_0^{1,000} = 666.67$

Expected losses up to 250:

$$\int_0^{250} \frac{x^2}{500,000}dx + 250\int_{250}^{1,000} \frac{x}{500,000}dx$$

$$= \frac{x^3}{1,500,000}\Big|_0^{250} + \frac{250x^2}{1,000,000}\Big|_{250}^{1,000} = 244.79$$

Expected losses up to 500:

$$\int_0^{500} \frac{x^2}{500,000}dx + 500\int_{500}^{1,000} \frac{x}{500,000}dx$$

$$= \frac{x^3}{1,500,000}\Big|_0^{500} + \frac{500x^2}{1,000,000}\Big|_{500}^{1,000} = 458.33$$

Expected losses at 250 deductible = 666.67 – 244.79 = 421.88
Expected losses at 500 deductible = 666.67 – 458.33 = 208.33
Indicated deductible relativity by moving to a \$500 deductible from the current \$250 deductible $= \frac{208.33}{421.88} = 0.494$

5.9

Limit	ILF	Cumulative Limited Loss Distribution
250,000	0.90	0.692
500,000	1.00	0.769
1,000,000	1.15	0.885
2,000,000	1.30	1.000

e.g. 0.885 = 1.15÷1.30

Percent of total losses expected to be paid by the reinsurer for the reinsurer layer \$500,000 excess of \$500,000 = 0.885 – 0.769 = 11.6%

5.10

Prior to Increase

Claim File ID	Total Amount of Claim	Amount Paid by Primary Insurer	Amount Paid by Reinsurer
1	250,000	250,000	0
2	495,000	495,000	0
3	540,000	500,000	40,000
4	750,000	500,000	250,000
Total	2,035,000	1,745,000	290,000

With 7% Increase on Every Claim

Claim File ID	Total Amount of Claim	Amount Paid by Primary Insurer	Amount Paid by Reinsurer
1	267,500	267,500	0
2	529,650	500,000	29,650
3	577,800	500,000	77,800
4	802,500	500,000	302,500
Total	2,177,450	1,767,500	409,950

$$\text{Increase to primary insurance company} = \frac{1,767,500}{1,745,000} - 1 = 1.3\%$$

$$\text{Increase to reinsurer} = \frac{409,950}{290,000} - 1 = 41.4\%$$

5.11 Losses above $500,000 and $2,000,000 can have different loss development patterns and trends. Also an attachment point of $2,000,000 implies that the covered policies have even higher limits. Companies or individuals that buy high limits usually have different risk profiles than those that buy lower limits, such as would be included with lower attaching treaties.

5.12 To stabilize its net loss ratio, an insurer wants to reduce its exposure to high severity or frequency events. It does this through the use of excess of loss or stop-loss reinsurance (high severity) or catastrophe reinsurance (high frequency). The reinsurer is taking on higher risk and therefore wants a higher return on its capital. As a result, the reinsurance premium will be greater than the present value of the expected losses. The result is that the primary insurer will forego some profits in return for limiting its potential losses.

5.13 Ceding commission ranges from 100% (60% loss ratio) to 0% (80% loss ratio). Using linear interpolation, at 75% loss ratio the ceding commission percentage = 3/4×0% + 1/4×100% = 25%. At 65% loss ratio, ceding commission percentage = 1/4×0% + 3/4×100% = 75%. Ceding commission at 65% loss ratio therefore is:

$$\$500,000 \times \frac{75\%}{25\%} = \$1,500,000.$$

5.14 Quota share treaty means that the primary insurer shares 60% of premium and 60% of all covered losses.

(a) Retained premium for primary insurer:
60%×(12,000 + 3,000 + 30,000) = $27,000

(b) Retained losses for the primary insurer:
60%×(150,000 + 50,000 + 300,000) = $300,000

5.15

Layer of Loss	Amount of $450 Loss in Layer	Percent Covered by Reinsurer	Amount Covered by Reinsurer
< $100	100	0%	0
$100 – $200	100	85%	85
$200 – $300	100	90%	90
> $300	150	95%	142.5
Total	450		317.5

Amount retained by primary insurer = $450,000,000 – $317,500,000 = $132,500,000